Number Patterns summary

- Terminology
- Types
- Notation and general formulae

Terminology

A *pattern* is a set of numbers of objects in which all the members relate to each other by a rule specific to that pattern.

A *number pattern* is a list or sequence of numbers that follow a certain rule.

Each number pattern consists of terms and each term has a value.

Examples:

-1; -2; -3; -4; ....

1; 3; 6; 10; ...

1; 3; 9; 27; ...

-3; -1; $-\frac{1}{3}$; $-\frac{1}{9}$; ...

1; 4; 9; 16; ...

1; 1; 2; 3; 5; 8; 13; ...

1; 10; -3; 39; ...

In a) the value of the first term is -1, the value of the second term is -2, the value of the third term is -3 and the value of the fourth term is -4.

If we were to write the sequence in example a) as:

-1 + (-2) + (-3) + (-4) + ...

it would be called a *series* as the terms of the sequence are being added.

Types of number patterns

Number patterns or sequences are categorised or named by the nature of the specific rule of that pattern.

Examples:

-1; -2; -3; -4; ....

This is a *linear number pattern* as the first difference is constant (-1).

It is also known as an *arithmetic sequence* due to the constant difference, in this case -1. By adding -1 to the previous term, you get the next term.

1; 3; 6; 10; ...

This is a *quadratic sequence* as the first difference is not constant but the second difference is constant.

Number	1		3		6		10
First difference		2		3		4	
Second difference			1		1		

1; 3; 9; 27; ...

This is a geometric sequence. It has a constant ratio of 3. To obtain the next term, you multiply the previous term by 3. It is neither a linear nor a quadratic sequence.

-3; 1; $-\frac{1}{3}$; $\frac{1}{9}$; ...

This is another geometric sequence. The constant ratio is $-\frac{1}{3}$.

It is neither linear nor quadratic.

1; 4; 9; 16; ...

This is a sequence of square numbers. It is also a quadratic sequence (check the first and second differences). It is neither arithmetic nor geometric.

0; 1; 1; 2; 3; 5; 8; 13; ...

This is known as the Fibonacci Sequence. It is a famous sequence that starts with 0 and is formed by adding the two previous terms. It is neither linear nor quadratic and neither geometric nor arithmetic.

1; 10; -3; 39; ...

This is just a random list of numbers. It has no mathematical rule that relates the numbers to each other and is therefore not a number pattern or sequence.

Notation and formulae

Example 1:

1; 2; 3; 4; ......

This number pattern or sequence has 4 terms.

The first term has a value of 1.

The second term has a value of 2.

The third term has a value of 3.

The fourth term has a value of 4.

The following mathematical notation is used to replace these statements:

$T_1 = 1$
$T_2 = 2$
$T_3 = 3$
$T_4 = 4$

This helps us to look for a relationship between the term number and its value. In this example they are equal. So the 20th term has a value of 20 and the n^{th} term has a value of *n*. We write this as:

$T_{20} = 20$
$T_n = n$

$T_n = n$ is also called the general term of this sequence as the variable *n* can be replaced with any number.

Example 2:

Write down the first four terms of the sequence:

$$T_n = \frac{2}{4n+1}$$

Here we are given the general term of this sequence.

If we want to find the first term we need to replace the *n* with a 1:

$$T_1 = \frac{2}{4(1)+1} = \frac{2}{5}$$

So the value of the 1st term is $\frac{2}{5}$

If we want to find the second term we need to replace the *n* with a 2:

$$T_2 = \frac{2}{4(2)+1} = \frac{2}{9}$$

If we want to find the third term we need to replace the n with a 3:

$$T_3 = \frac{2}{4(3)+1} = \frac{2}{13}$$

If we want to find the fourth term we need to replace the n with a 4:

$$T_4 = \frac{2}{4(4)+1} = \frac{2}{17}$$

The first four terms are therefore: $\frac{2}{5};\frac{2}{9};\frac{2}{13};\frac{2}{17}$

It is very important to understand the difference between the term number (T_n) and the term value (the answer when you substitute the value of n into the general term).

Example 3:

-4; -8; -12; -16; ...

The first term has a value of -4: $T_1 = -4$

The second term has a value of -8: $T_2 = -8$

The third term has a value of -12: $T_3 = -12$

The fourth term has a value of -16: $T_4 = -16$

a) What will the value of the 100th term be?

b) Write down T_n .

Looking at the pattern we can see that there is a constant difference of -4 (i.e. you add -4 to the previous term to get the next term).

a) So we could keep doing that till we find T_{100} . But this would take ages!

Instead we can look for the relationship between the term NUMBER and the term VALUE.

Notice that the value of each term is four times the term number.

Therefore $T_{100} = 4 \times 100 = 400$ and

$T_n = 4 \times n = 4n$

This type of sequence that has a constant difference is called an arithmetic sequence. The first term of an arithmetic sequence is usually denoted using an *a* and the constant difference with a *d.* The *n* is often used to represent the general term or any term in the sequence. So:

b) T_n indicates the general *term number.*

$4n$ is the general *term value.*

Example 4:

$3;5;7;9;...$

a) What type of sequence is this?

b) Calculate the 25th term.

c) Write down the general term of this sequence.

a) This term has a constant difference of 2. Therefore we know that:

It is an arithmetic sequence (it is also a linear sequence).

b) To find the 25th term we need to know the relationship or rule between the term number and the term value. Let's look at them:

$$T_1 = 3$$
$$T_2 = 5 = T_1 + 2 = T_1 + 1(2)$$
$$T_3 = 7 = T_1 + 2 + 2 = T_1 + 2(2)$$
$$T_4 = 9 = T_1 + 2 + 2 + 2 = T_1 + 3(2)$$

Look for a relationship between the term number and the term value.

In this case, the term number is one more than the value that the 2 is being multiplied by in the term value.

So to find the 25th term:

$$T_{25} = T_1 + 24(2) = 3 + 48 = 51$$

c) $T_n = T_1 + (n-1)2$

For convention purposes in mathematics, the first term of a sequence is symbolized by an a, the constant differene is denoted by a d and the term number is denoted by an n. So for this example:

$$T_1 = a$$
$$d = 2$$
$$T_n = a + (n-1)2$$

And the general term of any arithmetic sequence can be written as:

$$T_n = a + (n-1)d$$

Example 5:

4; 8; 16; 32; ...

The first term has a value of 4: $T_1 = 4$

The second term has a value of 8: $T_2 = 8$

The third term has a value of 16: $T_3 = 16$

The fourth term has a value of 32: $T_4 = 32$

a) What will the value of the 20th term be?

b) Write down T_n .

a) Looking at the pattern we can see that there is a constant ratio of 2 (i.e. you multiply the previous term by 2 to get the next term).

So we could keep doing that till we find T_{20} . But this would take ages!

Instead we can look for the relationship between the term NUMBER and the term VALUE.

$$T_1 = 4$$
$$T_2 = 8 = 4 \times 2 = T_1 \times 2 = T_1 \times 2^1$$
$$T_3 = 16 = 4 \times 2 \times 2 = T_1 \times 2 \times 2 = T_1 \times 2^2$$
$$T_4 = 32 = 4 \times 2 \times 2 \times 2 = T_1 \times 2 \times 2 \times 2 = T_1 \times 2^3$$

Do you notice a relationship between the exponent of the 2 in each case and the term number? (A difference of 1)

So $T_{20} = T_1 \times 2^{19} = 4 \times 524\ 288 = 2\ 097\ 152$

b) And $T_n = T_1 \times 2^{n-1}$

For convention purposes in mathematics, the first term of a sequence is symbolized by an *a*, the constant ratio is denoted by an *r* and the term number is denoted by an *n*. So for this example:

$$T_1 = a$$
$$r = 2$$
$$T_n = 4 \times 2^{n-1}$$

And the general term of any geometric sequence can be written as:

$$T_n = ar^{n-1}$$

Example 6:

a) Calculate $\sum_{k=0}^{4}(3k+1)$

b) What type of series is this?

This $\left(\sum\ \right)$ is called *sigma notation*. It simply means to "sum up" terms.

When we add terms of a sequence together, we call the sequence a *series.*

So the question is actually asking us to find the total of five terms of the sequence that has a general term of $T_k = 3k+1$.

a) In other words we want to find:

$T_0 + T_1 + T_2 + T_3 + T_4$

So we need to know what their values are first. To find their values, we must use the general term and substitution:

$$T_k = 3k + 1$$
$$T_0 = 3(0) + 1 = 1$$
$$T_1 = 3(1) + 1 = 4$$
$$T_2 = 3(2) + 1 = 7$$
$$T_3 = 3(3) + 1 = 10$$
$$T_4 = 3(4) + 1 = 13$$

Therefore we get: $1 + 4 + 7 + 10 + 13 = 35$

$$\sum_{k=0}^{4}(3k+1) = 35$$

b) This is an arithmetic series as it has a constant difference of 3.

Example 7:

Determine the 5th term of the geometric sequence of which the 8th term is 4 and the 12th term is 64.

Notation is crucial here! You need to translate this into mathematical notation:

$$T_8 = ar^7 = 4 \ldots\ldots\ldots\ldots(1)$$
$$T_{12} = ar^{11} = 64 \ldots\ldots\ldots\ldots(2)$$
$$T_5 = ar^4 = ?$$

We have two equations and two variables so we need to use simultaneous equation strategies. One of these strategies is to eliminate one of the variables.

If we divide equation 2 by equation 1 we can eliminate the *a*:

$$\frac{ar^{11}}{ar^{7}} = \frac{64}{4}$$
$$\frac{r^{11}}{r^{7}} = 16$$
$$r^4 = 2^4$$
$$r = 2$$

Substitute this back into equation 1 or 2:

$$ar^7 = 4$$
$$a \times 2^7 = 4$$
$$a = \frac{4}{128}$$
$$a = \frac{1}{32}$$

Therefore $T_5 = ar^4 = \frac{1}{32} \times 2^4 = \frac{1}{32} \times 8 = \frac{1}{4}$